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# Critical dynamics in Heisenberg ferromagnets and antiferromagnets near the percolation threshold

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**Abstract.** The critical spin-wave dynamics of dilute ferromagnetic and antiferromagnetic Heisenberg systems near the percolation threshold is studied on a two-dimensional regular (fractal) model for bond percolation, which captures the relevant geometric features of the cluster structure. The dynamic critical exponent and spectral dimension for both the ferromagnetic and antiferromagnetic systems are calculated. The treatment of the dynamics in these systems requires the use of generalised scaling techniques for sublattice systems, which involve an extension of the parameter space. Giving an interpretation to the extended parameter space for the systems considered and exploiting the multiparameter scaling in the equations of motion, we present an explicit derivation of a relation between the dynamic exponent and static exponents, namely the bond conductance exponent and the fractal dimension of the percolating network, for ferromagnetic systems. This provides a new method of relating the dynamic and static exponents via dynamic scaling alone, in contrast to previous forms based on crossover arguments and the use of relationships between physical quantities.

## 1. Introduction

This paper is concerned with the critical spin dynamics of dilute ferromagnetic and antiferromagnetic Heisenberg systems near the percolation threshold. Although static dilution-induced critical phenomena have long been studied (for a review see Stinchcombe (1983a)), much less is known about critical dynamics in dilute systems (Harris and Kirkpatrick 1977, Korenblit and Shender 1978, Stinchcombe 1985).

Dilution induces criticality in spin systems at zero temperature for concentrations $p$ in the neighbourhood of the percolation threshold $p_c$, where the percolative correlation length $\xi$ diverges. For diverging $\xi$, holes on all length scales appear on the infinite cluster, which becomes scale invariant. The length scaling of the mass of the cluster is then characterised by a fractal dimension $d_f$, such that $M(L) \sim L^{d_f}$, where $d_f < d$ and $d$ is the embedding Euclidean dimension. Dynamical processes become critical on such a self-similar structure. In particular, the dynamics of (long wavelength) spin waves is then described by an anomalous power-law dependence of frequency $\omega$ on 'wavevector', or inverse characteristic length $k$ characterised by a dynamic exponent $z$. In fact, the divergence in $\xi$ induces a crossover in the dynamics of spin waves on the percolating network, from hydrodynamic behaviour for $k\xi < 1$ to critical behaviour for $k\xi > 1$. In the case of dilute Heisenberg ferromagnets the dispersion relation for

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spin-wave excitations in the hydrodynamic regime ($k\xi > 1$) is given by $\omega = D(p)k^2$, where the spin-wave stiffness $D(p)$ depends on the concentration $p$, while in the critical regime ($k\xi > 1$) it takes the form $\omega \propto k^z$. It has been shown (Brenig *et al* 1971, Kirkpatrick 1973) that $D(p)$ is directly related to static properties of the percolating cluster, namely the percolative conductivity and the percolation probability. Assuming that the crossover between the two asymptotic regimes develops continuously in the crossover region, $k\xi \sim 1$, leads to a scaling law which relates $z$ to static percolative exponents (Harris and Stinchcombe 1983). The behaviour of dilute Heisenberg antiferromagnets can be similarly discussed using appropriate forms for the spin-wave dispersion in those systems (Harris and Kirkpatrick 1977, Christou and Stinchcombe 1988).

Much attention has been given to the study of the geometrical properties of dilute systems near $p_c$, which is crucial for fully understanding the physical properties of these systems (for reviews on percolation see Stauffer 1979, Essam 1980). Various geometrical models have been proposed to imitate the infinite incipient cluster at the percolation threshold, and it is of great interest to understand the effects of these different geometries on the magnetic properties. In particular, for the backbone of the infinite cluster which results from eliminating the dangling ends (and is the relevant object for the discussion of, for example, propagation of correlations and flow of current) three different pictures have been presented, namely the 'nodes and links' picture (Skal and Shklovskii 1975, de Gennes 1976), the 'nodes, links and blobs' picture (Stanley 1977), and the Sierpinski gasket model (Gefen *et al* 1981). The 'nodes, links and blobs' picture is currently accepted as the most realistic (Coniglio 1982).

Recently there has been considerable interest in regular fractal models for percolating clusters at criticality, the main reason being that the exact solution of these models gives insight into the physical properties of the real (random) systems. Mandelbrot (1984) and Mandelbrot and Given (1984) have presented fractal models for the infinite cluster at the percolation threshold, which capture the essential geometric features. However, these models do not describe the approach towards the threshold. For above the percolation threshold it is believed that on length scales smaller than the percolative correlation length, the backbone of the infinite cluster is self-similar, consisting of 'nodes' joined by singly connected 'links' and multiply connected 'blobs', but on larger length scales it becomes homogenous. Nagatani (1985) has proposed a regular model in two dimensions for bond percolation just above the threshold, which incorporates all the previously mentioned features for the infinite cluster and, furthermore, gives the distribution of finite clusters, allowing the derivation of explicit expressions for the quantities characterising the approach towards the threshold. In this paper we will study the critical spin-wave dynamics of Heisenberg systems in this regular model. The model, in addition, has the advantage of providing a picture for the full cluster and not only for the backbone (unlike other models) which is important for the treatment of the dynamics in which the dangling ends are relevant. The values for the critical bond concentration, the fractal dimensionality of the infinite cluster and of its backbone, the correlation length and the conductivity exponents, and also the scaling form of the cluster distribution, obtained for the model, are in very good agreement with those of random percolation (Nagatani 1985). The model is thus expected to provide an adequate basis for the study of the dynamics.

The dynamics of randomly diluted spin systems near $p_c$ has been considered by Stinchcombe (1983b) and Harris and Stinchcombe (1983) who applied real space renormalisation group methods respectively to the dynamics of a dilute ferromagnetic

chain and of a two-dimensional hexagonal lattice ferromagnet. This required, however, the use of averaging methods to deal with the random equations of motion, leading to approximate recursion equations for the scaling parameters, namely the frequency and the concentration. In turn, non-random fractal models can be solved exactly by the real space methods since these methods involve a decimation process which is the inverse of that used in the construction of the fractals, namely recursive insertions of a structure on a smaller scale. Harris and Stinchcombe (1983) have calculated the dynamics of a Heisenberg ferromagnet on a regular fractal model, the Sierpinski gasket. The treatment of the dynamics of fractals with sublattices, of which the Nagatani model is an example, does, however, require the use of generalised scaling techniques (Stinchcombe and Maggs 1986) involving an extension of the scaling parameter space. An interpretation will be given to the extended parameter space of the Heisenberg systems studied here, which we will exploit to explicitly derive a relationship between the dynamic exponent and static exponents, namely the bond conductance exponent and the fractal dimension, for ferromagnetic systems. As mentioned before, a scaling law relating the dynamic exponent to static percolative exponents for dilute ferromagnets has been obtained (Harris and Stinchcombe 1983) by combining dynamic crossover arguments with the use of a relationship between physical quantities. In contrast, our derivation shows that it is possible to relate the dynamic and static exponents in an explicit form by dynamic scaling alone.

This paper is organised as follows. In § 2 we calculate the critical spin-wave dynamics of Heisenberg ferromagnets and antiferromagnets in the regular model for bond percolation discussed, applying scaling techniques for sublattice systems. In § 3 we present a derivation of a relationship between the dynamic exponent and static exponents for ferromagnets, based on an interpretation of the extended parameter space arising in the dynamic scaling of the sublattice systems. Finally, in § 4 we summarise the results in this paper.

## 2. Critical spin-wave dynamics in dilute ferromagnetic and antiferromagnetic Heisenberg systems

We now study the dynamics of ferromagnetic and antiferromagnetic Heisenberg systems on the two-dimensional regular model for bond percolation proposed by Nagatani. Figure 1 illustrates the fractal structure of the model, from which the critical dynamics arises. This structure can be obtained by recursively replacing each bond by the same generator of the Mandelbrot-Koch curve (Mandelbrot and Given 1984) and has a fractal dimension $d_f = \ln 8/\ln 3$.

The Hamiltonian describing Heisenberg spin systems is

$$H = -\sum_{ij} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j \tag{1}$$

where the exchange couplings $J_{ij}$ between the spins $\boldsymbol{S}_i$ are positive in the case of ferromagnets, $J_{ij} > 0$, and negative in the case of antiferromagnets, $J_{ij} < 0$. We consider nearest-neighbour couplings only and assume that the spins are normalised to unity. A scaling method is used to calculate the dynamic critical exponents for the systems considered, which is based on the transformation of the equations of motion under a length scaling achieved by elimination of sites (decimation).

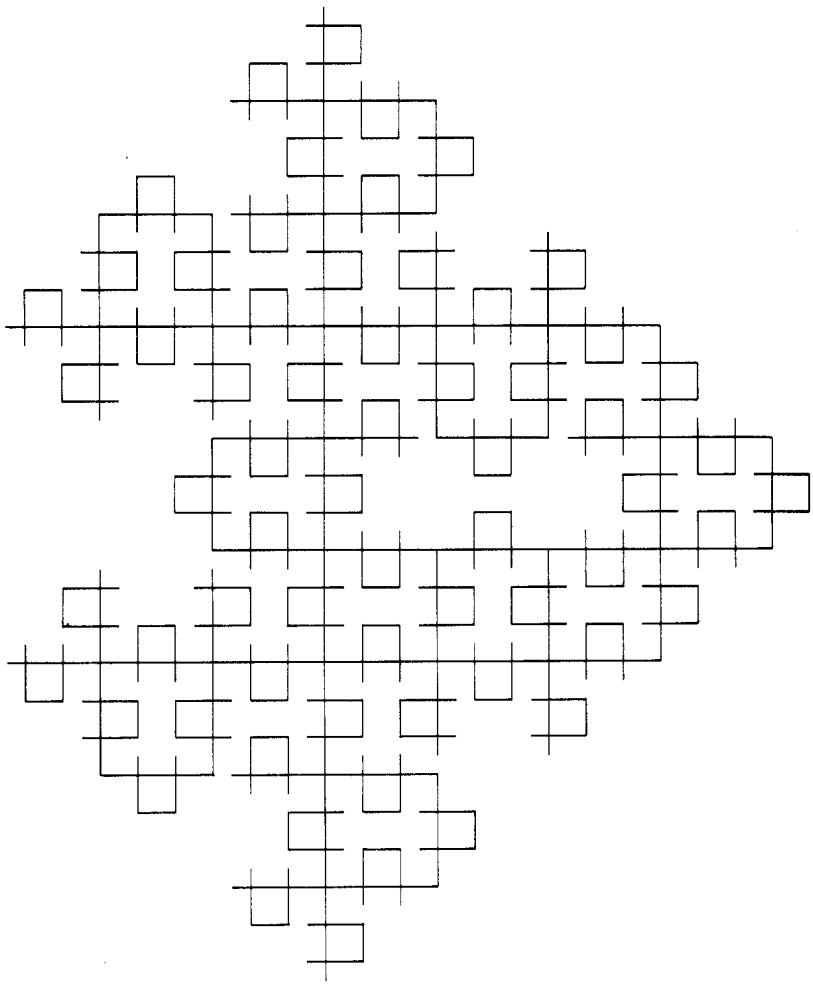

**Figure 1.** Fractal lattice in Nagatani's model for bond percolation.

(i) *Ferromagnet.* The linearised equations of motion governing the transverse spin dynamics of a Heisenberg ferromagnet at zero temperature can be written in the form

$$(n_i u - \omega) S_i^{\top} = \sum_{j=1}^{n_i} S_j^{\top} \tag{2}$$

where $S_i^{+}$ is the usual combination $S_i^{+} = S_i^{x} + iS_i^{y}$ of the transverse spin components at site $i$, $\omega$ is the frequency divided by coupling constant $J$, and the sum is over the $n_i$ nearest neighbours of $i$. The Goldstone symmetry in the problem is restored by imposing the condition $u = 1$. Now, it should be noted that in the fractal lattice considered here, there are three types of sites according to the number of nearest neighbours $n_i$, $n_i = 1$, 2, 3. Because of this multiplicity of non-equivalent sites on the lattice, the original form of the equations (2), where $u = 1$, is not maintained under scaling and hence needs generalisation. It turns out that the parameters $u$ and $\omega$ are both sufficient and

necessary to perform an exact scaling, having the initial conditions $u = 1$ with $\omega$ arbitrary; an extension of the parameter space from one parameter ($\omega$) to two ($\omega$, $u$) is thus required.

Eliminating sites on the original lattice as illustrated in figure 2, for a lattice rescaling by a scale factor $b = 3$, leads to the following scaling transformations for the parameters $\omega$ and $u$:

$$\omega' = \frac{\omega a}{(2u-\omega)^2(u-\omega)^2} \qquad u' = \frac{u(a-b)+\omega b}{(2u-\omega)^2(u-\omega)^2} \tag{3}$$

where

$$a = [(4u-\omega)(2u-\omega)(u-\omega)-(2u-\omega)]^2$$
$$-[(2u-\omega)(u-\omega)+(4u-\omega)(u-\omega)-1]^2$$
$$b = [(4u-\omega)(u-\omega)-1][(2u-\omega)^2-1]-(2u-\omega)(u-\omega).$$

The linearised form of these equations at the fixed point $(\omega^*, u^*) = (0, 1)$ has eigenvalues $\lambda = \frac{11}{4}$, 22 with associated scaling fields given by (to first order in the variables $\omega$ and $\delta = (u-u^*))\omega$, $\omega - \frac{7}{3}\delta$, respectively. The larger of the eigenvalues determines the low-frequency scaling of $\omega$ since the initial symmetry conditions ($u = 1$, $\omega$ arbitrary) do not cause the vanishing of the associated scaling field which remains linear in $\omega$. This yields for the dynamic critical exponent of the ferromagnetic system the value

$$z_F = \ln 22/\ln 3 \simeq 2.81. \tag{4}$$

As shown by Rammal and Toulouse (1983) the spectral dimension $d_s$ characterising the scaling of the low-frequency density of spin-wave states in the infinite cluster can be related to the dynamic critical exponent $z$ and the fractal dimension $d_f$ by $d_s = z/d_f$. So (4) implies for the spectral dimension of the ferromagnetic infinite cluster the value $d_s = \ln 8/\ln 22 \simeq 0.67$. This is in good agreement with the result found by Lewis and Stinchcombe (1984) in a direct numerical computation of the density of states of a two-dimensional randomly diluted system at the percolation threshld, and with the (approximate) conjecture of Alexander and Orbach ($d_s = \frac{2}{3}$) (Alexander and Orbach 1982).

(ii) *Antiferromagnet.* For an antiferromagnet on such a fractal lattice there are again three types of sites according to the number of nearest neighbours, but now each of these sites can be occupied either by a spin up or a spin down. This implies that

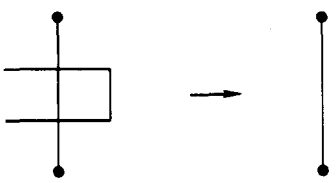

**Figure 2.** Scaling of a bond in a ferromagnetic fractal lattice.

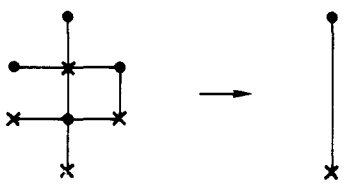

**Figure 3.** Scaling of a bond in an antiferromagnetic fractal lattice (● spin up, × spin down).

the exact scaling of the antiferromagnet requires a generalisation of the equations of motion involving three parameters, which are now written in the form

$$(-n_i u-\omega)S_i^+ = \sum_{j=1}^{n_i} S_j^+ \tag{5}$$

if at site $i$ there is a spin up and

$$(-n_i v+\omega)S_i^+ = \sum_{j=1}^{n_i} S_j^+ \tag{6}$$

if at site $i$ there is a spin down. The initial conditions for the scaling are now $u=v=1$ with $\omega$ arbitrary.

Eliminating sites on the antiferromagnetic lattice as shown in figure 3, for a lattice rescaling by a scale factor $b=3$, gives the following recursion relations for the parameters $\omega$, $u$, $v$:

$$\omega' = \frac{\omega a}{(2u+\omega)(2v-\omega)(u+\omega)(v-\omega)} \qquad u' = \frac{u(a-b)-\omega b}{(2u+\omega)(2v-\omega)(u+\omega)(v-\omega)}$$

$$v' = \frac{v(a-c)+\omega c}{(2u+\omega)(2v-\omega)(u+\omega)(v-\omega)}$$

with

$$\begin{aligned} a = {} & [(2u+\omega)(2v-\omega)-1][(4u+\omega)(v-\omega)-1][(4v-\omega)(u+\omega)-1] \\ & -(2u+\omega)(v-\omega)[(4v-\omega)(u+\omega)-1] \\ & -(2v-\omega)(u+\omega)[(4u+\omega)(v-\omega)-1]-(2u+\omega)(2v-\omega)(u+\omega)(v-\omega) \end{aligned}$$

$$b = [(2u+\omega)(2v-\omega)-1][(4u+\omega)(v-\omega)-1]-(2u+\omega)(v-\omega)$$

$$c = [(2u+\omega)(2v-\omega)-1][(4v-\omega)(u+\omega)-1]-(2v-\omega)(u+\omega).$$

The eigenvalues of these equations linearised at the fixed point $(\omega^*, u^*, v^*)=(0, 1, 1)$ are $\lambda = 1, \frac{11}{4}, 22$. The associated linear scaling fields (to first order in the variables $\omega$, $\delta=(u-u^*)$, $\eta=(v-v^*)$) are respectively $\omega-\frac{7}{2}(\delta-\eta)$, $\omega$, $\delta+\eta$. Due to the symmetry $\delta=\eta=0$ present in the original system of equations, the scaling field associated with the largest eigenvalue vanishes in linear order, so higher-order approximations to the scaling fields are needed to extract the dynamic exponent $z$. A calculation to second order shows that the scaling field $\delta+\eta+\frac{23}{11}\delta\eta-\frac{6}{11}(\delta\omega-\eta\omega)+\frac{25}{44}(\eta^2+\delta^2)-\frac{29}{44}\omega^2$ associated with the eigenvalue $\lambda=22$ is initially of order $\omega^2$. It follows then that the antiferromagnetic dynamic critical exponent is given by

$$z_A = \tfrac{1}{2}\ln 22/\ln 3 \simeq 1.41. \tag{7}$$

Introducing this result in the Rammal-Toulouse relation gives for the spectral dimension of the antiferromagnetic infinite cluster the value $d_s = 2\ln 8/\ln 22 \simeq 1.35$.

It turns out that the results (4) and (7) for the ferromagnetic and antiferromagnetic dynamic exponents of this particular fractal satisfy the same relationship $z_{AF}=\frac{1}{2}z_F$ as observed in pure or simple systems (see also Stinchcombe and Maggs 1986). This relationship is not expected to apply in random cases (see Maggs and Stinchcombe 1986).

## 3. Derivation of a relation between the dynamic and static exponents by a scaling approach

The scaling of systems with sublattices usually requires an extension of the parameter space appearing in the equations of motion, as observed in the previous section. An interpretation of the extended parameter space arising for the case of the magnetic systems considered is here given which provides insight into the scaling process. It will be shown that from the multiparameter scaling in the equations of motion it is possible to obtain both dynamic and static exponents. Based on this, we will derive a scaling relation between the dynamic and static exponents for ferromagnetic systems.

If in the equations of motion for the ferromagnet (2) one writes $u = 1 + h/J$, these will be

$$(n_i J + h_i - w) S_i^+ = J \sum_{i=1}^{n_i} S_j^+ \tag{8}$$

where $w$ is now simply frequency. It turns out then that (8) is formally identical to the equations of motion for a ferromagnet in a longitudinal site-dependent magnetic field $h_i$ such that $h_i = n_i h$ at site $i$. In a similar way for the antiferromagnet, if one writes $u = 1 + \hat{h}/J$ in (5) and $v = 1 - \check{h}/J$ in (6) the equations obtained are formally identical to those of an antiferromagnet in a longitudinal magnetic field with magnitude $\hat{h}_i = -n_i \hat{h}$ or $\check{h}_i = n_i \check{h}$ according to whether the spin at site $i$ is up or down. The initial conditions for scaling are then for the ferromagnet $h_i = 0$ and for the antiferromagnet $\hat{h}_i = \check{h}_i = 0$. Thus one can interpret the extension of the parameter space for both the ferromagnet and the antiferromagnet as follows. Under decimation systems originally in zero magnetic field scale into systems with effective applied fields produced by the decimated spins. The magnitudes of the fields on each site are proportional to the coordination number of the sites, since such fields result from equal contributions from each of the neighbouring decimated bonds. For the antiferromagnet one naturally expects the field acting on the up spins to be of opposite sign to that acting on the down spins.

Given this interpretation of the extended parameter space in terms of effective magnetic fields, it turns out that the scaling field associated with the eigenvalue that determines the dynamic exponent for the ferromagnet considered in § 2 is of the form (in linear order) $(w - \bar{h})/J$ where $\bar{h} = \bar{n} h$ is the average of the fields on the different types of sites on the lattice and $\bar{n}$ is the average coordination number. Since $h$ is an effective field arising from the removal of bonds, one expects that at $w = 0$ it scales in the inverse way to the number of bonds, i.e. that $h' = b^{d_f} h$. Also, one observes that at $w = 0$ and $h = 0$ equation (8) becomes formally equivalent to Kirchoff's law for a network of conductances $\sigma = J$ (connecting potentials $V_i = S_i^+$), and hence under such conditions ($w = 0$, $h = 0$) the scaling of the coupling $J$ becomes identical to the scaling of the bond conductance, i.e. $J' = b^{-\hat{t}/\nu} J$ where $\hat{t}$ is the conductance exponent and $\nu$ the correlation length exponent. So, in principle it should be possible to relate the dynamic critical exponent which characterises the scaling of the field $(w - \bar{h})/J$ to the static exponents characterising the scaling of the parameters $h$ and $J$ involved in the field, namely the fractal dimension $d_f$ and the conductance and correlation length exponents, $\hat{t}$ and $\nu$ respectively.

The scaling of the coupling $J$ and the field $h$ can be obtained from the transformation of (8) under decimation, fixing $w$ to be the same in the original and scaled systems. For simplicity we consider here the scaling of the backbone of the infinite cluster which

involves only two types of sites with two and three nearest neighbours. The resulting scaling transformations for the parameters $h$ and $J$ at fixed $w$ are

$$J' = J^3 \frac{(2J+2h-w)^2}{(2J+2h-w)^2(3J+3h-w)^2 - J^2(5J+5h-2w)^2}$$

$$h' = h + J - J^2 \frac{(2J+2h-w)^2(3J+3h-w) - J^2(5J+5h-2w)}{(2J+2h-w)^2(3J+3h-w)^2 - J^2(5J+5h-2w)^2}.$$

The linearised form of these equations at the fixed point $(J^*, h^*) = (0, 0)$ and $w = 0$, is given by

$$J' = \tfrac{4}{11}J - (288/11^2)h + (116/11^2)w \tag{9a}$$

$$h' = 6h - 2w. \tag{9b}$$

From (9*a*) it turns out that indeed at $w = 0$ and $h = 0$ the scaling of the coupling is given by the same eigenvalue as the scaling of the bond conductance which for this particular lattice is $b^{-\hat{t}/\nu} = \frac{4}{11}$. Also, from (9*b*) it follows that at $w = 0$ the eigenvalue characterising the scaling of the field is the inverse of that for the number of bonds, which in the case of the backbone is $b^{d_l} = 6$.

Now, in linear order and at fixed $w$, the scaling field associated with the eigenvalue, which determines the ferromagnetic dynamic exponent for the backbone, scales like

$$\left(\frac{w}{J'} - \frac{5}{2}\frac{h'}{J'}\right) = b^z \left(\frac{w}{J} - \frac{5}{2}\frac{h}{J}\right). \tag{10}$$

Then introducing (9*a*) and (9*b*) into (10), one finds that $z$ is related to $\hat{t}/\nu$ and $d_f$ by the form

$$z = \hat{t}/\nu + d_f. \tag{11}$$

This relationship leads to the form of the scaling law relating $z$ to the static exponents for the conductivity and the percolation probability of dilute ferromagnets derived by Harris and Stinchcombe (1983) ($z = 2 + (t-\beta)/\nu$)), by using the relation between the percolation probability exponent and the fractal dimension ($\beta = d - d_f$) and relating the conductivity ($t$) and conductance ($\hat{t}$) exponents ($(t - \hat{t}) = (d-2)\nu$).

## 4. Conclusions

A regular fractal model for bond percolation in two dimensions was used to investigate the critical spin-wave dynamics of dilute ferromagnetic and antiferromagnetic Heisenberg systems near the percolation threshold. Although missing disorder aspects, and also exhibiting a discrete scale invariance while the real random fractals are statistically scale invariant, the model captures the relevant geometric features of the cluster structure near the threshold. The dynamic critical exponent and the spectral dimension for the ferromagnetic and the antiferromagnetic systems were calculated. The treatment of the dynamics in these systems required the use of generalised scaling techniques for sublattice systems which involve an extension of the parameter space. Giving an interpretation to the extended parameter space and exploiting the multiparameter scaling in the equations of motion, we presented an explicit derivation of a relation between the dynamic critical exponent and static exponents, namely the conductance exponent and the fractal dimension, for ferromagnetic systems. This provides an alternative method of relating dynamic to static exponents via dynamic scaling alone.

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